Math 195: Demystifying Mathematics

Homework 6 : Due April 10

As always, I am mainly looking for good, well articulated ideas and complete sentences of explanation for (2) and (3) below. If you're stuck on the math of any of these problems, remember you are welcome to email me to set up an appointment to meet. Also, I assume you've read through the notes posted on PWeb for Mar 30-31 and Apr 1-2. (Note there is a problem (3) on the back.)

1. An intro problem which is just a chance to compute something. Compute Soundex code for the last names: Smith, Schmid, Smyth, and Schmidt. Show me your work for each of these four names, but you don't have to add a lot of words to explain this problem.

2. It turns out that ISBN-10 numbers (which we talked about before break and which I wrote about on the April 1-2 class notes) are able to *correct* one transposition error between two adjacent numbers in most cases. So not only can you detect an error has happened by seeing that the weighted sum is not a multiple of 11, but you can also figure out exactly where the error happened, assuming you know the error was a transposition of adjacent numbers. In this problem you will explore how to do that correction.

   (a) Let's work backwards. We will introduce transpositions into a valid number and see how that disrupts the weighted sum. The ISBN Number $0-192-69067-1$ is a valid 10-digit ISBN number. Verify that this is indeed a valid ISBN number and carefully record the sum you got (which better be a multiple of 11). You can just show me your work with few words.

   (b) Now transpose 2 **adjacent** digits of the ISBN number in (a) **NOT including the first or last digit** and compute the weighted sum. (For example, maybe you swap the 2 and first 9 to get the number $0-129-69067-1$ which is not a valid ISBN number.) You can write your computations down for me without additional words. But do make sure to somehow show me how you got the value.

   (c) Next pick 3 more adjacent numbers from the ISBN number in (a) and swap them and compute the weighted sum like you did in (b) for each transposition separately. Again, show me the work you did and final answer.

   (d) Now between (b) and (c) you have four examples. Consider the numbers you got for the sums in those 4 examples. How do they compare to the sum in (a)? What is the connection between these sums and the numbers you intentionally "swapped" in (b) and (c)? This part may take some time staring at the numbers looking for patterns or clues.

   (e) Articulate (i.e. explain with words) the strategy based on your answer to (d) for correcting an incorrect ISBN number if the error was a swap of two adjacent numbers. Then use your strategy to correct the incorrect ISBN number $0-669-03925-4$ formed by transposing two adjacent digits of a correct ISBN number.

3. You may have asked yourself why numbers like 3 and 7 and 9 were used as weights for the UPC or Bank Routing Numbers. It turns out that in order to successfully detect all single digit errors, we need to choose weights which are **relatively prime to** 10 (remember the word relatively prime means two numbers have a gcd of 1). We won't prove this directly but you can see why some numbers that are not relatively prime to 10 won't always work to detect every single digit error. I hope you'll agree that 2 and 5 are not relatively prime to 10.

   (a) Make up a 11 digit UPC number and weigh it with alternating 2 and 1 instead of 3 and 1. Create a single error and see if you are able to detect the error with the 2 and 1 weighting . Keep going until you make a single digit error that appears to give you a correct UPC code even though it was not the UPC you started with.

   (b) If the weights 5, 1, 5, 1, ..., 5, 1 were used for a UPC code, what single-digit errors would go undetected?