Math 218: Combinatorics

Homework 5 : Due September 20

1. (a) Suppose L is the list of elements $[a, b, b, c, c]$. In class we discussed how to count all possible permutations of this list. Now suppose we want to choose 4 elements from L and that the order they are chosen matters. How many ways can you do this for the particular list L?

   (b) Generalize your argument to create an algorithm to choose $n-1$ objects in a list L with a_1 of the element x_1, a_2 of the element x_2, a_3 of the element x_3 and so on up to a_k of the element x_k, when order matters.

2. How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 40$$

   if we also need to assume $x_1 \geq 2$, $x_2 \geq 0$, $x_3 \geq -5$, and $x_4 \geq 8$? (Hint: Can you do a change of variables that makes this like a problem we already know how to solve?)

3. Show that if $n+1$ integers are chosen from the set $\{1, 2, \ldots, 2n\}$ then there are always two which differ by 1.

4. Prove that if the average of n non-negative integers m_1, m_2, …, m_n is greater than $r-1$, then at least one of the integers is greater than or equal to r.