Math 218: Combinatorics

Homework 4 : Due September 15

For problems that ask for combinatorial reasoning, you may either discuss the problem in terms of counting sets, or make it "real world" by connecting the identity to a particular counting problem which the values count.

1. Determine the number of circular permutations of $\{0, 1, 2, \ldots, 9\}$ in which 0 and 9 are not opposite each other.

2. (a) Use *combinatorial reasoning* to prove that for n and k positive integers, $k\binom{n}{k} = n\binom{n-1}{k-1}$.
   (b) Use (a) and Example 4.2.4 in Morris to prove the following equality in a different way than you did on the last homework.
$$1 + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \cdots + \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1} - 1}{n+1}.$$

3. Use *combinatorial reasoning* to prove the identity for $1 \leq k \leq n$
$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}.$$

4. Use *combinatorial reasoning* to prove the following identity for $0 \leq k \leq \frac{n}{2}$.
$$\sum_{m=k}^{n-k} \binom{m}{k}\binom{n-m}{k} = \binom{n+1}{2k+1}.$$
   (Small hint: $2k + 1$ is an odd number.)

5. A bakery sells 6 different kinds of pastry. If the bakery has at least a dozen of each kind, how many different options are there for a box of a dozen pastries?