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# Math 218: Combinatorics

HOMWORK 16 : DUE DECEMBER 1

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1. Morris 12.2.6 #1 (basically) Prove that given the set of vertices  $V$  of a graph  $G$ , the relation on all  $v_1, v_2 \in V$

$$v_1 \sim v_2 \text{ if } v_1 \text{ and } v_2 \text{ are connected}$$

is an equivalence relation.

2. Prove that a simple graph of order  $n \geq 2$  always has two vertices of the same degree
3. Prove that any two connected graphs of order  $n$  with degree sequence  $(2, 2, \dots, 2)$  are isomorphic. (Hint: Use the definition of isomorphism of graphs and create the  $\phi$  in the definition.)
4. There are 11 non-isomorphic simple graphs of order 4.
- (a) Draw one representative of each isomorphism class. If you  $\text{\LaTeX}$ , check out the “Add a Picture” instructions on PWeb.
- (b) For this problem, which cases does problem 3 help us classify?
- (c) Pick two other degree sequences not in (b) and not the trivial graph (with no edges) and prove that all graphs of that degree sequence are isomorphic to the given representative.
5. (a) Prove that a simple graph of order  $n$  with at least

$$\frac{(n-1)(n-2)}{2} + 1$$

edges must be connected.

- (b) Give an example of a disconnected simple graph of order  $n$  with one fewer edge.