Math 218: Combinatorics

Homework 11 : Due November 1

1. In the previous assignment we gave a formula for $S(n, n-2)$. Here we prove a similar result but for signless Stirling numbers of the first kind.

   Prove for all $n \geq 3$ that
   $$c(n, n-2) = 2 \cdot \binom{n}{3} + 3 \cdot \binom{n}{4}.$$

2. In how many ways can eleven pieces of identical candy be passed out to four children so that no child gets more than four pieces (and some children could get 0 pieces)?

3. Let n be a positive integer and k some integer so that $n + 1 \leq k \leq 2n$.

   (a) What is the number of permutations of $[2n]$ whose longest cycle is of length k?

   (b) Answer the question in (a) if $k = n$ instead.

4. Bogart # 166. Prove that the number of partitions of k into even parts (meaning each part is an even number) and the number of partitions of k into parts of even multiplicity are the same. (Even multiplicity means each value of the parts shows up an even number of times.)

5. As Bogart defines in section 3.3.4, the number of partitions of k into n parts that are each distinct from each other is denoted by $Q(k, n)$. Prove that $Q(k, n) = Q(k-n, n) + Q(k-n, n-1)$.