Math 218: Combinatorics

1. For both parts of this question, be sure to explain how you arrived at your answer.

    (a) Find the number of compositions of 10 into even parts.

    (b) Find the number of weak compositions of 25 into 5 odd parts.

2. For $n \geq 2$, $S(n, 2) = 2^{n-1} - 1$.

    (a) Prove this using induction.

    (b) Prove this using a combinatorial proof.

3. Prove $S(n, n-2) = \binom{n}{3} + 3\binom{n}{4}$ for $n \geq 3$.

4. Suppose you run a pet sitting service with 4 employees, and you currently have 12 dogs and 1 cat to care for. (Assume the dogs are indistinguishable.)

    (a) How many ways can you assign the pets to your employees?

    (b) How many ways can you assign the pets to your employees if each employee must be assigned at least 1 pet?